MEASUREMENT OF JOINT PROBABILITY IN TURBULENT DISPERSION OF HEAT FROM TWO LINE SOURCES

by

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SUMMARY

Resistance thermometer correlation measurements of the thermal wakes behind two line heat sources are suggested for estimation of the (Lagrangian) joint probability density of fluid particle displacements. Considerable uncertainty results in practice from the concomitant molecular heat diffusion.

Experiments indicate that in addition to the well-known nearly normal (Gaussian) shape of the probability densities of individual particle displacements, these displacements are also close to being jointly normal.

INTRODUCTION

If we mark with dye a particular portion of a turbulent flow, we shall see the marked fluid change position and shape as time progresses. These two changes are manifestations of the diffusive character of the flow. If we repeat the experiment in a "statistically identical" flow, the changes experienced by the marked fluid will in general be different, but if the experiment is repeated often enough certain average quantities can be defined. The same will be true in a single flow with statistical properties stationary in time. For example, the mean square displacement of the fluid mass from the mean path line and the average increase of a dimension of the marked fluid volume are two such quantities characterizing the diffusion. It is the goal of turbulent diffusion theory and experiment to be able to predict such (Lagrangian) measures of diffusion from other properties of the flow field, preferably those more easily measured.

When the contaminant does not change the dynamical properties of the marked fluid, this problem is obviously kinematical (in the absence of molecular transport): knowledge of the properties of path lines in the fluid would be sufficient to determine the turbulent convection.

Taylor[1] suggested that even with molecular transport, the obvious independence of random molecular migration and random turbulent migration permits simple superposition of the two resulting mean square displacements, hence the two transport effects. Townsend[2] has recently disproved this conclusion on the grounds that the obvious distorting effect of turbulent convection on local scalar gradients actually changes the local molecular transport rate.

The problem of diffusion in a homogeneous turbulent flow with no molecular diffusion was treated by Taylor in 1921[3]. He related the simplest measure, the mean square displacement of a single fluid particle as a function of time, to the double Lagrangian time correlation of velocity. So far no one has been successful in predicting this correlation function from the general dynamics of the flow or in relating it to the (more accessible) Eulerian properties. Its traits are known only through experiment. Another pertinent function, the probability density of particle displacement in a homogeneous turbulence, has been shown experimentally to be normal at all diffusion times[4, 5, 6], but no theory has been able to deduce this fact.

At the present time, therefore, we have available Taylor's theoretical connection between diffusion and Lagrangian correlation [written in more general form by Batchelor[7]] plus the empirical evidence for a normal probability density of particle displacement in shear-free homogeneous turbulence. By superposition of point source results these are sufficient for prediction of the average contaminant concentration field due to any prescribed spatio-temporal distribution of source strength.

Taylor's approach is applicable only in a homogeneous turbulence, and the most detailed data are available in the approximately homogeneous (and isotropic) turbulence behind a regular grid in a wind tunnel. Our resulting semi-empirical understanding supplies us at least with a jumping off point for conjecture and for inspection of the diffusion in more complex "practical" turbulent flows*.

*See, for example, "Remarks on Turbulent Heat Transfer" by S. Corrsin, Proc. Iowa Conference on Thermodynamics, April 1953.

When more detailed information is desired, for example the probability that the concentration at a fixed space point will exceed a chosen value with more then one active source, superposed single source data are inadequate; we must know the joint probability function for the particle displacements from the several sources.

The simplest generalization of the fairly well-studied single line source case is that of two parallel line sources. The simplest turbulence available is the decaying approximately isotropic turbulence behind a grid. The most convenient contaminant is heat. The present work was undertaken to establish an experimental method for measuring the joint probability function for such an arrangement.

This joint dispersion problem has been analytically described in some detail by Batchelor[8] who also treated special aspects theoretically[9]. In both works he excluded molecular diffusion effects. The paper of Brier[10] should also be noted.

The problem and experimental technique developed here were suggested by Professor S. Corrsin, who also supervised the investigation. Miss V. O'Brien carried out much of the calculation.

THE PROBLEM OF THE LINE SOURCE IN TURBULENT FLOW WITH MOLECULAR DIFFUSION

With the usual average space-time transformation used in isotropic turbulence experiments $\left[\frac{d}{dt} \doteq \bar{U} \frac{d}{dx} \right]$, the spread of the region contaminated by the marked particles with increasing downstream distance from the source is a measure of their lateral diffusion. This diffusion is not exclusively a result of the turbulence, however, since even if the turbulence level were zero there would be a spread due to molecular effects. If there were no molecular diffusion, the marked particles would constitute a surface with wrinkles which tend to increase with time, hence with downstream distance. The general spatial region occupied intermittently by this wavey sheet we call the "turbulent wake". The local region surrounding the particles marked at the source, where the "marking" diffuses by molecular motions, will be called the "molecular wake"; hence this is the instantaneous wrinkled sheet as thickened by molecular transport. The total region influenced by the "marking", irrespective of the diffusive mechanism, will be called the "mean wake".

The word "wake" here is used to denote the region contaminated by the source. The fluid particles defining this region are marked in some detectable fashion, in this case with heat. The sources and the contaminant are assumed to have negligible effect on the dynamical properties of the fluid, as restriction satisfied by the actual experiments; the momentum wake was undetectable 1/2 inch behind the source wire.

In accordance with a suggestion of Taylor(1), the two mechanisms causing diffusion are usually assumed to be uncorrelated in the sense that the mean square width of the mean wake is taken to be equal to the sum of the mean square widths of the turbulent and the undisturbed molecular wakes. Townsend(2) has recently pointed out that they are actually correlated in the following sense: the molecular diffusion is not independent of the turbulent motion because the random turbulent strain convectively change the instantaneous temperature gradients in such a way that the average molecular wake width after a given diffusion time is greater than that of the molecular wake in a non-turbulent flow.

If, however, it is found that the actual average molecular wake width is the same regardless of its centerline location (i.e., of its position within the turbulent wake), then Taylor's assumption can be reinvoked in modified form: the turbulent dispersion is uncorrelated with the accelerated molecular dispersion. Hence the squares of the corresponding standard deviations add to give the squared standard deviation of the mean wake:

$$\sigma^2 = \sigma_T^2 + \sigma_m^2$$

In Taylor's original suggestion[1] σ_m was taken from the non-turbulent molecular wake.

In this investigation the above lack of correlation is established experimentally. A theoretical proof would involve showing zero correlation between the turbulent particle displacement $y(t) = \int_0^t v(t_1)\, dt$, and whatever function of strain rate [$\sim$ velocity spatial derivative] is proportional to the increase in average molecular wake width.

Townsend's accelerated molecular diffusion as described above is easily estimated only for turbulence spectral components (eddies?) with wave length much larger than the molecular wake width. This permits approximation by a spatially constant strain field*. For spectral components with wave length

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* See the analysis in Appendix 2 of reference 2.

much smaller than the molecular wake a different viewpoint seems appropriate: these fluctuations spread the molecular wake from "within" just as the molecular motion does. Then the term "molecular wake" loses whatever appropriateness may have remained. The effective diffusivity of this very small scale motion could be estimated with the analysis of Corrsin[11] for heat transfer in isotropic turbulence with prescribed mean temperature gradient.

For turbulent fluctuations of scale comparable with the molecular wake width neither of the above simplified accounts is adequate, and a full-fledged non-uniform strain analysis would have to be carried out.

ANALYTICAL REPRESENTATION

Taylor's now classical analysis of the diffusion from a single line source in homogeneous, non-decaying turbulence with no molecular diffusion shows that the mean square spread of the turbulent wake is given by[3]

$$\frac{d\overline{y^2}}{dt} = 2\,\overline{v^2}\int_0^t S(\tau)\,d\tau = 2\,\overline{v^2}\int_0^t \frac{\overline{v(t)\,v(t-\tau)}}{\overline{v^2}}\,d\tau$$

where $y = \int_0^t v(t_1)\,dt_1$ is particle displacement. v is fluctuation velocity in the y-direction.

Perhaps the main problem in single source diffusion theory is the prediction of $S(t-\tau)$, the Lagrangian time correlation, in terms of the more easily measured Eulerian statistical functions. The analysis leading to the above requires the turbulence to be stationary in time. Therefore diffusion in decaying turbulence can be fitted into this simple mold only if it obeys some suitable similarity conditions. Townsend[2] has, in effect, assumed complete similarity of all functions both Eulerian and Lagrangian. This assumption collapses the diffusion data remarkably well considering that there seems to be no a priori reason to expect such detailed self-preservation of the decaying turbulence field.

This gives a result for $S(\tau)$ approximately independent of the location of the source in a given decaying field. Before the calculation can be carried out the molecular influence has to be removed from the $\overline{y^2}$ data, which he did by measuring the accelerated molecular diffusion and by assuming it uncorrelated to the turbulent diffusion.

If the functional form of the probability density of the displacement is characterized by one parameter (such as its standard deviation) then a knowledge of the Lagrangian correlation function gives a fairly complete statistical picture of the diffusion. In "isotropic" wind tunnel turbulence this probability density function is observed to be normal (Gaussian) at all diffusion times so that knowledge of the Lagrangian correlation function permits the mean concentration to be computed at a given point for any source distribution.

It is pertinent to examine the time variation of temperature at a fixed point in the wake. Near the source this time variation is very spiky with a fixed variation between the ambient temperature (say zero) and some peak value.[6] This peak is fairly insensitive to velocity gradients, so that any observed variation in peak temperature at the fixed point is caused by the Molecular wakes not being completely swept past the point. Near the edge of the turbulent wake the temperature is zero except for infrequent pulses due to the molecular wake's entering this region. Near the center of the turbulent wake the pulses are more frequent and the temperature only rarely drops to the zero value. After greater diffusion time the signal has essentially the same character, but now there is no clearly defined maximum temperature since the instantaneous peak value is determined by the time history of the turbulent and laminar effects on the diffusion, and the longer the time the larger the variability. For example, after great diffusion time the skewness of the temperature fluctuation at a fixed space point is considerably reduced[6].

The source configuration considered in this paper is that of two parallel line sources at the same downstream distance from the turbulence producing grid. The problem has two parameters in the sense that the spacing of the sources and the distance downstream from the sources are the variables used in exploring the joint statistical properties of the wakes. For both zero source spacing and large source spacing the problem reduces to an application of single source results, since for spacings large compared to all length scales of the turbulance the wakes move independently.

The function of interest in this investigation is the joint probability density of particle displacements from the two sources. As will be shown shortly, this is approximately proportional to a measurable quantity, the correlation between temperature fluction signals at two points in the turbulent wakes. This relationship is the essence of the experimental technique introduced here.

In particular, if the stream direction is x, the direction perpendicular to the main stream and the line sources is y, and the direction parallel to the sources is z, then the function of interest is the probability, defined as a time average that one turbulent wake is contained in a region $(x, y_1, z) \leftrightarrow (x, y_1+\Delta y, z)$ and simultaneously the other turbulent wake is contained in a region $(x, y_2, z) \leftrightarrow (x, y_2+\Delta y, z)$ (Fig. 2). In the experiments the x-location of the sources is constant, say zero, and all measurements are made at the same z. Therefore, the probability is a function of the source spacing Y, the distance from the source x, and the two lateral positions, y_1, y_2. This function is denoted by

$$P(y_1, y_2 ; Y, x)$$

where y_1 and y_2 are the individual y-displacements of the two wakes.

By *a priori* reasoning it is possible to make certain statements about this function. Near the source the probability density function of the displacements is identical to that of the velocities at the two points if we apply the appropriate space-time transformation. Therefore, all known statistical properties should be recoverable from the displacement measurements, e.g. the double correlation and the skewness of the velocity difference. Within a reasonable distance from the sources, for a fixed z, the two wakes cannot cross, and hence,

$$P(y_1, y_2; Y, x) = 0 \quad \text{for} \quad y_2 - y_1 > Y$$

The transverse isotropy of the flow also imposes the condition that

$$P(y_1, y_2; Y, x) = P(-y_2, -y_1; Y, x)$$

The remaining restriction is that the single wake should be recoverable from the marginal distribution. If the single wake probability density for the displacement is $p(y;x)$, then

$$p(y_1;x) = \int_{-\infty}^{\infty} P \, dy_2$$

and

$$p(y_2;x) = \int_{-\infty}^{\infty} P \, dy_1$$

EXPERIMENTAL EQUIPMENT

The experimental work was carried out in a 2 x 2 foot N.P.L. type open return wind tunnel at a mean velocity of 15 feet per second (Fig. 1). The turbulence was generated by a bi-plane square mesh grid with 1-inch mesh and 1/4-inch round rods.

Two spring loaded, vertical, .005-inch, heated Nichrome wires which spanned the tunnel served as sources for marking the fluid particles. The wires were mounted 20 mesh lengths from the grid in a frame which permitted independent parallel motion in the y-direction. The wires were heated with direct current to a temperature of approximately 300° C, this temperature producing no measurable dynamical effects on the heated fluid at the position where measurements were taken. There was no observable sag or motion of the wires due to aerodynamic forces.

The instantaneous temperature was detected downstream of the sources by two platinum resistance thermometers .00005 inch in diameter, constructed from Wollaston wire. The thermometers were operated with sufficiently low current ($<$ .001 amp) so that the velocity output was negligible compared to the temperature output at the x-locations used[12]. These thermometers were mounted on a traversing mechanism that permitted independent motion in the y-direction, i.e. perpendicular to the mean flow direction and to the sources. The y-location with respect to the source location could be established within .002 inch.

Mean temperatures were measured by averaging the temperature signal with long period galvanometer. One thermometer was used in the wake system, and the other was placed in the stream outside the region of wake influence. Large scale temperature fluctuations affecting both wires were cancelled by a bridge circuit. A Leeds and Northrup K-2 potentiometer was used to measure the voltage.

The instantaneous wake temperatures were converted to usable voltages by amplifying and compensating the thermometer output in accordance with ordinary hot-wire anemometry practice. The thermometer had a time constant of .1 millisecond and the compensated voltage output was flat to within 2% from 2 to 8000 cycles per second.

MEASUREMENT OF THE JOINT PROBABILITY OF THE WAKE DISPLACEMENTS

The joint probability of particle displacements from the two sources is essentially the fraction of total time that the wakes are simultaneously at two specified y -locations. Since our thermal tagging diffuses molecularly, correction must be made if we seek the turbulent dispersion only. Toward this end we need a measurement of the average "thickness" of this hot sheet for each position in the x, y plane. In fact we should like an average "local" temperature distribution for each such position. This can be measured approximately in a single wake via the correlation between temperature fluctuations picked up by two resistance thermometers (cool "hot-wires") separated in the y -direction.

Let $f(y; x, t)$ be the single-wake temperature field. The semi-colon is simply a reminder that we focus attention upon $f(y)$ for fixed x and t. $f(y)$ can be described also as a function of the y -location of its mean (y') and the distance from this mean ($y - y'$), i.e.

$$f(y) = f_1 (y-y'; y', x, t)$$

where

$$y' \equiv \frac{\int_{-\infty}^{\infty} y f(y)\, dy}{\int_{-\infty}^{\infty} f(y)\, dy}$$

For each value of y' (and x, of course) there is an average temperature distribution $\tilde{f}_1 (y-y'; y', x)$; this is the one we should like to determine experimentally for the purpose of correcting the gross mean wake. In principle it could be determined as a time average of

$f_1(y-y';\, y', x, t)$ with y' held fixed, i.e. the average of the collection of instantaneous f's which occur whenever the mean position of the hot sheet sweeps past a chosen y'.

In order to make an experimental estimate of $\tilde{f}_1$ we assume (a) that $\check{f}_1$ is a normal (Gaussian) curve, though lower and broader than the normal thermal wake which would occur with no turbulence; (b) that the instantaneous $f(y)$ profiles differ only a small amount from the "conditional mean function" $\tilde{f}_1(y-y')$. Under these assumptions, the directly measurable temperature correlation function,

$$\underline{R(r,y)} \equiv \overline{f(y)\, f(y+r)}$$

is approximately equal to the hypothetical correlation that would result from random lateral translation (by variation of y') of a fixed profile $\tilde{f}_1(y-y')$.

$$R(r,y) \approx \int_{-\infty}^{\infty} \tilde{f}_1(y-y')\, \tilde{f}_1(y+r-y')\, p(y')\, dy'$$

where $p(y')$ is the probability density of y', i.e. of the turbulent wake.

With assumption (a) plus the known closely normal character of $p(y')$ we get

$$R(r,y) \approx \frac{1}{(2\pi)^{3/2}\,\sigma_p} \int_{-\infty}^{\infty} \frac{1}{\sigma_f^2(y')}\, e^{-\frac{(y-y')^2}{2\sigma_f^2}} \cdot e^{-\frac{(y+r-y')^2}{2\sigma_f^2}} \cdot e^{-\frac{y'^2}{2\sigma_p^2}}\, dy'$$

and we have assumed $\sigma_f(y' \pm r) \approx \sigma_f(y')$ for small r, where σ_f and σ_p are the standard deviations of $\tilde{f}_1$ and p.

What we want to check is whether σ_f is independent of y, as assumed by Townsend[2]. Since the independence assumption works fairly well for single wake measurements, σ_f can be at most a slowly varying function of y. In particular, if the variation of σ_f is small over a distance comparable with σ_f, whence the major contribution to the integral comes, then we can write approximately

$$R(r,y) \approx \frac{\exp\left\{-\frac{1}{2\sigma_f^2}\left[2y^2 + 2ry - r^2 - \frac{\sigma_p^2(2y+r)^2}{2\sigma_p^2+\sigma_f^2}\right]\right\}}{2\pi\sigma_f\sqrt{2\sigma_p^2+\sigma_f^2}}$$

Presuming that the measured, $R(r,y)$ does give normal $R(r)$ for each y, and that σ_p is known, the above expression could be used in principle to calculate $\sigma_f(y)$. In fact, σ_p is not known a priori but we do have σ_θ, the standard deviation of the mean thermal wake. For this calculation we must therefore estimate σ_p; the simplest estimate is perhaps

$$\sigma_p^2 \approx \sigma_\theta^2 - \langle \sigma_f^2 \rangle_{ave.}$$

where $\langle \sigma_f^2 \rangle_{ave.}$ is the average value of $\sigma_f^2(y)$ across the mean wake, at fixed x.

A straightforward way to use this statistical information on f for determining $P(y_1, y_2; Y, x)$ is to measure the double correlation between the temperature fluctuations in the two wakes, the measurements being taken at the same x. The result of such a measurement would be a function of y_1, y_2, the thermometer positions. This function is

$$G(y_1, y_2; Y, x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{f}_1(y_1 - y_1'; y_1', x)\, \tilde{f}_1(y_2 - y_2'; y_2', x)\, P(y_1', y_2'; Y, x)\, dy_1'\, dy_2'$$

This expression assumes that $f(y_1)$ is independent of $f(y_2)$ [in the second wake] at the same instant. $\tilde{f}_1$ is defined as an average, and this assumption means that the average shape of f_1 at a particular y is independent of a subsidiary condition such as the fact that nearly fluid particles have also been tagged.

Unfortunately, inversion of the above expression for P is simple only for special cases, such as $\tilde{f}_1$ being a Dirac type function; hence, this approach is not advisable if others are available. Furthermore, the standard deviations of $\tilde{f}_1$ and p in the experimental configuration used happened to be of the same order of magnitude, which makes $\tilde{f}_1$ far from a Dirac function.

Since the information sought requires measurement of the location of the fluid which has actually been "in contact" with the sources, a method was devised that used only the peak values of the temperature. The turbulence keeps the molecular wake symmetric on the average, so that the peak temperature is located at the symmetry point, i.e. the point that was "in contact" with the source.

The manner in which the measurements were performed was to make the temperature at a point actuate a trigger circuit in such a way that when the temperature exceeded a certain level, a constant non-zero voltage was produced which went again to zero as soon as the temperature dropped below this level. With two channels of this type, one for each wake, signals are generated which yield a fixed voltage when either transducer is in a region where the wake exceeds a given temperature. These binary signals, fluctuating between zero and a fixed value not zero, depending on the absence or presence of the hot regions of the wakes, when multiplied together yield a signal whose integral is proportional to the desired joint density.

This type of operation on the temperature signal converts each molecular wake into a rectangular shape whose width δ is determined by the width of the molecular wake at the temperature used to actuate the trigger. If this rectangle is called $g(y-y', y'; x)$, the number obtained by multiplying together the signals from both wakes is given by

$$G_g(y_1, y_2; Y, x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(y_1-y_1', y_1'; x)\, g(y_2-y_2', y_2'; x)\, P(y_1', y_2'; Y, x)\, dy_1' dy_2'$$

If the critical temperature is high enough so that the variation of P is small in the region δ, then

$$G_g \sim P(y_1', y_2'; Y, x)\, \delta_1 \delta_2 + \cdots\cdots$$

Using the signals in the manner indicated here requires that an accurate reproduction of the temperature signals be available to actuate the trigger. The hot-wire amplifiers were a.c. coupled for reasons of stability, so that the output signal from the amplifier had a zero mean value. In order to regain a voltage actually proportional to the instantaneous temperature, it is necessary to regain the mean value. This was done by either clamping the signal to the ambient temperature level of the signal, i.e. making zero voltage correspond to the ambient tunnel temperature, in locations where a well-defined zero existed (near the edges), or by adding a voltage proportional to the mean temperature in regions where no base level existed (wake center).

These signals were shaped, as described above, with Schmitt trigger circuits into on-off signals. The trigger signals from each wake were instantaneously multiplied and the resulting coincidence signal was integrated to give a measure of the joint probability distribution. The integration was accomplished by using the coincidence signal to modulate a regular pulse train and counting those pulses that passed when the signal was non-zero. A diagram of the circuit that performs the necessary operation is shown in figure (3).

In order to determine the correct voltage to be added to the signal for restoring the mean temperature value, measurements of the wake of a single line source were made with the same method used for the two wakes, except that here the integration was performed on g instead of the coincidence signal from the two wakes. The result of this measurement is equivalent to

$$H(y) = \int_{-\infty}^{\infty} g(y-y';y';x)\, p(y';x)\, dy' \approx p(y;x)\cdot\delta + O(\delta^2)\cdots$$

$p(y')$ was obtained from the mean wake and the experimentally determined $\tilde{f}_1(y-y')$. Sufficient voltage was added at each point of the signal to make $H(y)$ proportional to $p(y)$. In general, appreciable addition of voltage was needed in only the central portion of the turbulent wake.

RESULTS

Measurements were made at the distances of x = 7/8, 6-7/8, and 18-7/8 inches from the sources.

The function $\overline{f(y)\, f(y+r)}$ was measured in order to obtain information about the molecular wakes necessary to adjust the equipment and to interpret the results. The measured points, normalized on $\overline{f(0)\, f(0)}$, are shown in figures 4, 5, and 6. Assuming a Gaussian shape and making use of the fact that the (accelerated) molecular wakes and the turbulent wakes are approximately uncorrelated, we obtained values of σ_f from these curves in conjunction with the mean temperature curves. The assumptions involved in this analysis are apparently not wholly satisfied, since the value of σ_f obtained depends on the particular points of the correlation function used for its computation. An average value of σ_f was taken at each y, and these are plotted in figure 7. The values of σ_f show no consistent variation with y at a particular x, and within the scatter they are constant, indicating independence of the two diffusion phenomena. The fact that the ratio σ_f/σ_p is approximately unity for all diffusion times considered is surprising, and presumably a coincidence for the particular experimental conditions.

The ~~mean wake~~ at x = 18-7/8 inches is shown in figure 8 as typical of these measurements. Also shown is a comparison of the turbulent wake as computed from the mean wake and as measured directly with the trigger circuit. The circuit was used here with the signal clamped to its lowest value, i.e. the minimum value attained by the signal over a time interval of 1/2 second was used as the base line voltage. Near the edges of the wake the circuit clamps on to the ambient temperature and the two curves agree closely. When the joint probabilities were measured, the circuit was adjusted so that the output corresponded to the correct turbulent wake, as explained in the preceding section.

The joint probability functions were obtained by measuring a set of conditional probability curves and then using these to construct a contour plot of the desired functions. This plot was normalized to unity at its peak value. A typical set of measured conditional probability curves is shown in figure 9. The contours were constructed from interpolation of such data. The contours were symmetrical around $y_1 = -y_2$ in all cases, as required by isotropy, so that only those on one side of $y_1 = -y_2$ were plotted.

Near the sources, where the Lagrangian velocity correlation coefficient is close to unity, the measured joint probability densities of the displacements should be identical with the velocity joint probability densities at the two source points (with the appropriate scaling). The measured joint probability densities of the displacements at 7/8 inches (figures 10 and 11) were therefore used to calculate the double correlation of the velocities at the same location, and these values are plotted in figure 12 together with the directly measured velocity correlation curve. The computed points give a smaller correlation coefficient than the direct measurement, a circumstance that may be attributable to the Lagrangian correlation's not being sufficiently close to one at the x-location used.

When molecular diffusion has been in action so long that the molecular wakes spread into each other for an appreciable range of $(y_1 - y_2)$, the measuring technique fails since the wakes become indistinguishable. In this region the joint probability function is not completely recoverable. Because of this reduced reliability the measurements made where this effect was dominating are not reported here. These include measurements at $X = 6\text{-}7/8$ inches, 18-7/8 inches, and $Y = 1/4$ inch.

The data for $Y = 3/4$ inch (figures 11, 13, 14) illustrate the propagation of the joint probability density for particle displacement in time. The particle motions remained correlated for all diffusion times observed, as indicated by the non-zero eccentricity of the elliptical contours. At $X = 18\text{-}7/8$ the flattening of the contours near $y_2 - y_1 = Y$ is a manifestation of the constraint forbidding particle paths to cross.

From the contour plots at $Y = 3/4''$, the marginal distributions of $(y_1 - y_2)$ were computed and are shown in figure 15. For this particular source spacing and at these diffusion times, this function stays similar and is approximately Gaussian in shape.

The marginal distribution of y_1 and y_2 computed from the probability hills did not correspond exactly to the directly measured distribution of y_1 or y_2. This difficulty is due to the lack of precision at low values of the probability, and possibly to a lack of independence between the turbulent and molecular wakes.

The growth of $\overline{(y_1 - y_2)^2}$ as a function of X (9) is shown in figure 16.

CONCLUSIONS

This study of measuring technique for determining the joint probability density of particle displacements in a turbulent flow field discloses several effects that have to be correctly accounted for in any further work along these lines. When fluid particles are marked with a substance that can diffuse by molecular motion, the resulting finite size of the convected region enters the measurements in such a fashion that the laminar and turbulent effects are inseparable. Near the sources, given y values for the wake locations imply a value of the velocity derivative. This same velocity gradient will change the widths of the molecular wakes, and this change is reflected in any measurements weighted by the wake widths at a given value of the temperature. The large shift of the peak value of the probability hill from the geometrical center ($y_1 = y_2 = 0$) in the present measurements illustrates this effect most noticeably. The shift had been entirely attributed to the known skewness of the velocity difference in isotropic turbulence until it was seen that the marginal distributions of y_1 and y_2 computed from these data also had the peaks shifted. Assuming that the wakes were acted on by the same velocity gradient for all diffusion times considered, and estimating the velocity derivatives at each wake, yields a result for the peak shift in rough agreement with the measured value which indicates that the analysis has a proper basis.

The results show further more that the average shape of the molecular wake at a given diffusion time is independent of y , if no other condition is specified, so that the assumption of uncorrelated molecular and turbulent effects is usable in the single source case.

The observed changes in time for the Y = 3/4 inch joint probability densities show that the joint probability density function does not stay similar in time but requires more than one parameter for its description. This seems reasonable because the constraints controlling the behavior of the function at various diffusion times are different. Near the source the displacement joint probability distribution is similar to the velocity joint probability distribution, while far from the source the fact that the wakes cannot cross becomes significant.

Finally, it is suggested that studies for long diffusion times (when molecular broadening has "overlapped" the two wakes in these air experiments) be carried out in water, which has a considerably higher Prandtl number.

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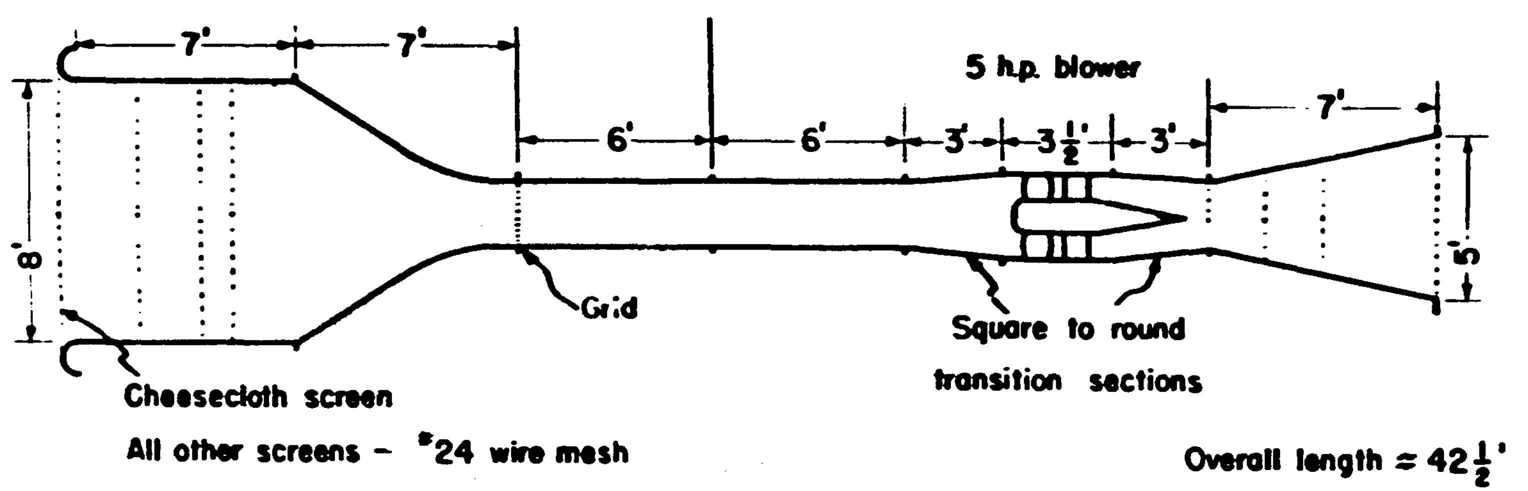
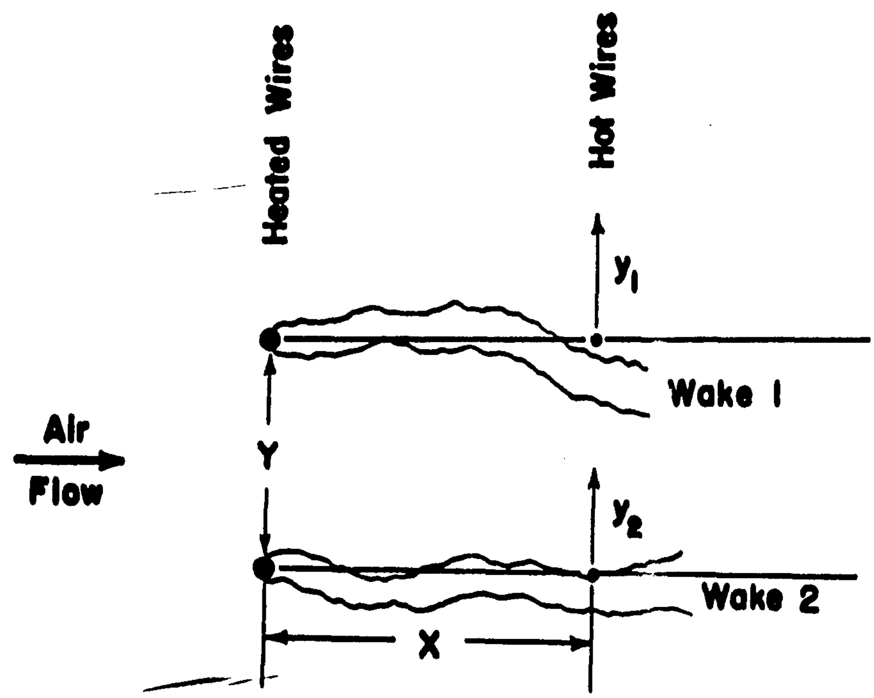
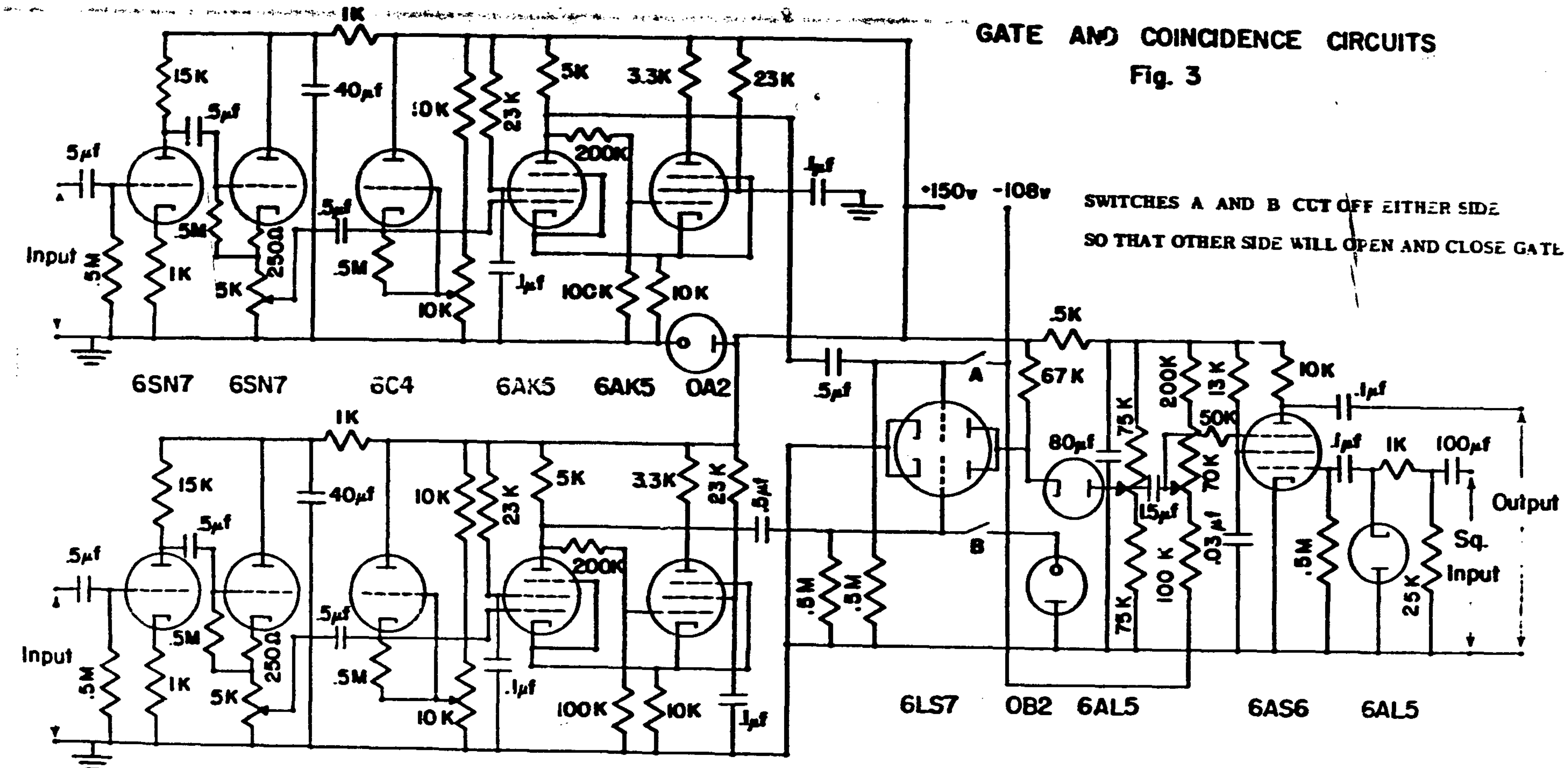

7'
7'
6'
6'
3'
3 1/2'
3'
7'
5 h.p. blower
8'
5'
Grid
Square to round
transition sections
Cheesecloth screen
All other screens - #24 wire mesh
Overall length ≈ 42 1/2'

Fig. 1



Geometrical Arrangement

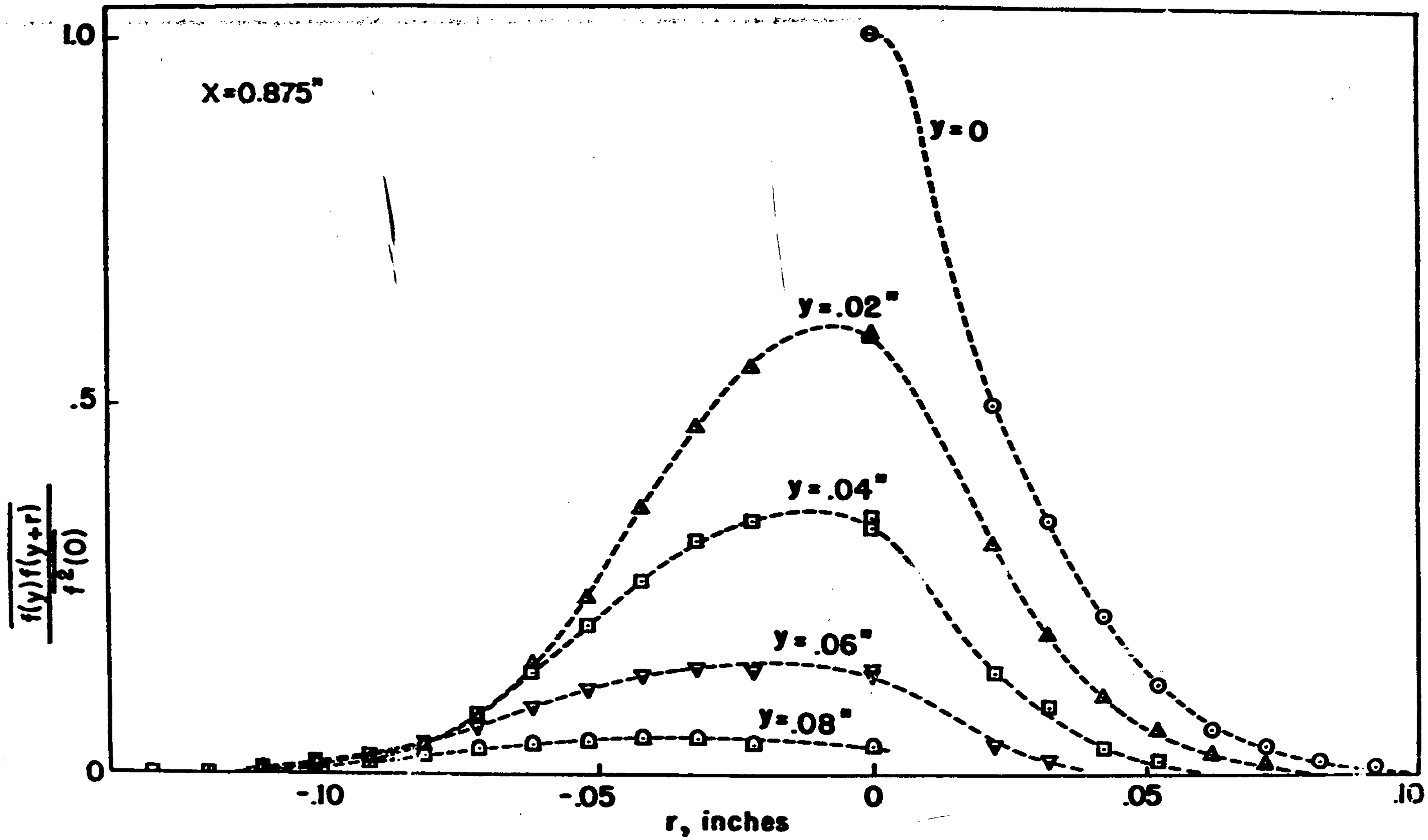
Fig. 2



GATE AND COINCIDENCE CIRCUITS

Fig. 3

SWITCHES A AND B CUT OFF EITHER SIDE SO THAT OTHER SIDE WILL OPEN AND CLOSE GATE



X=0.875"
y=0
y=.02"
y=.04"
y=.06"
y=.08"
1.0
.5
0
-.10
-.05
0
.05
.10
r, inches
$\frac{f(y)f(y+r)}{f^2(0)}$

Fig. 4

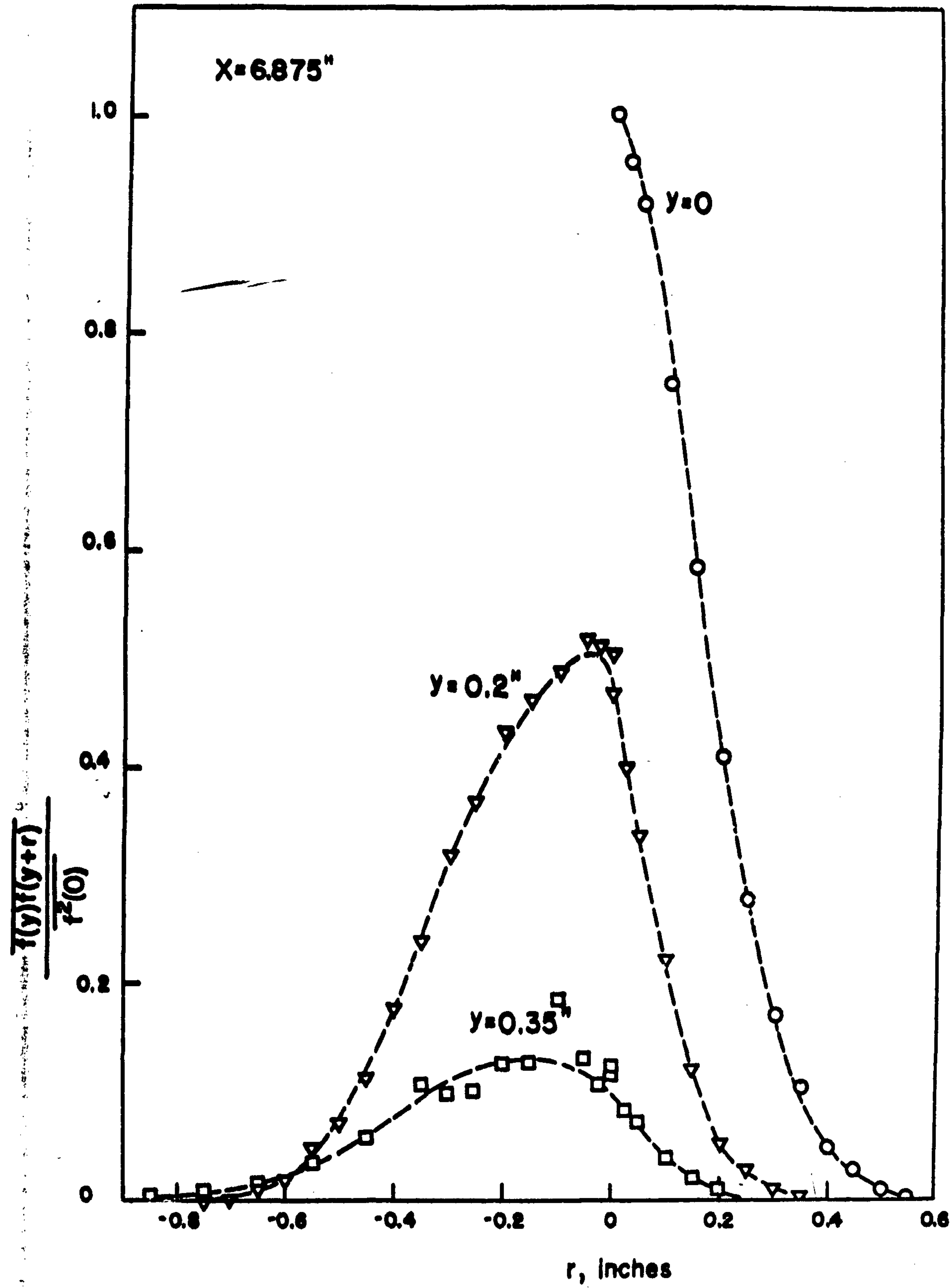
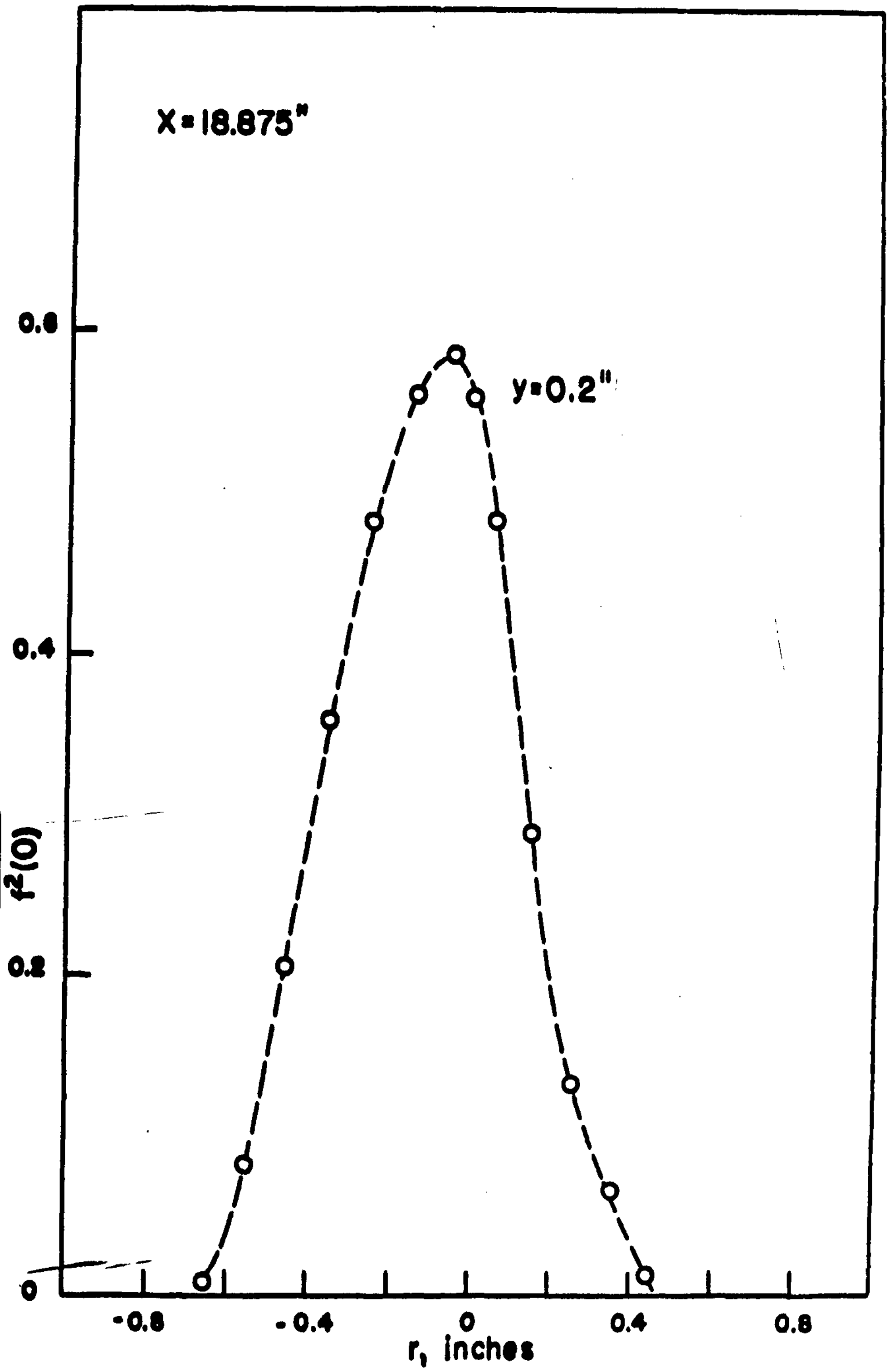
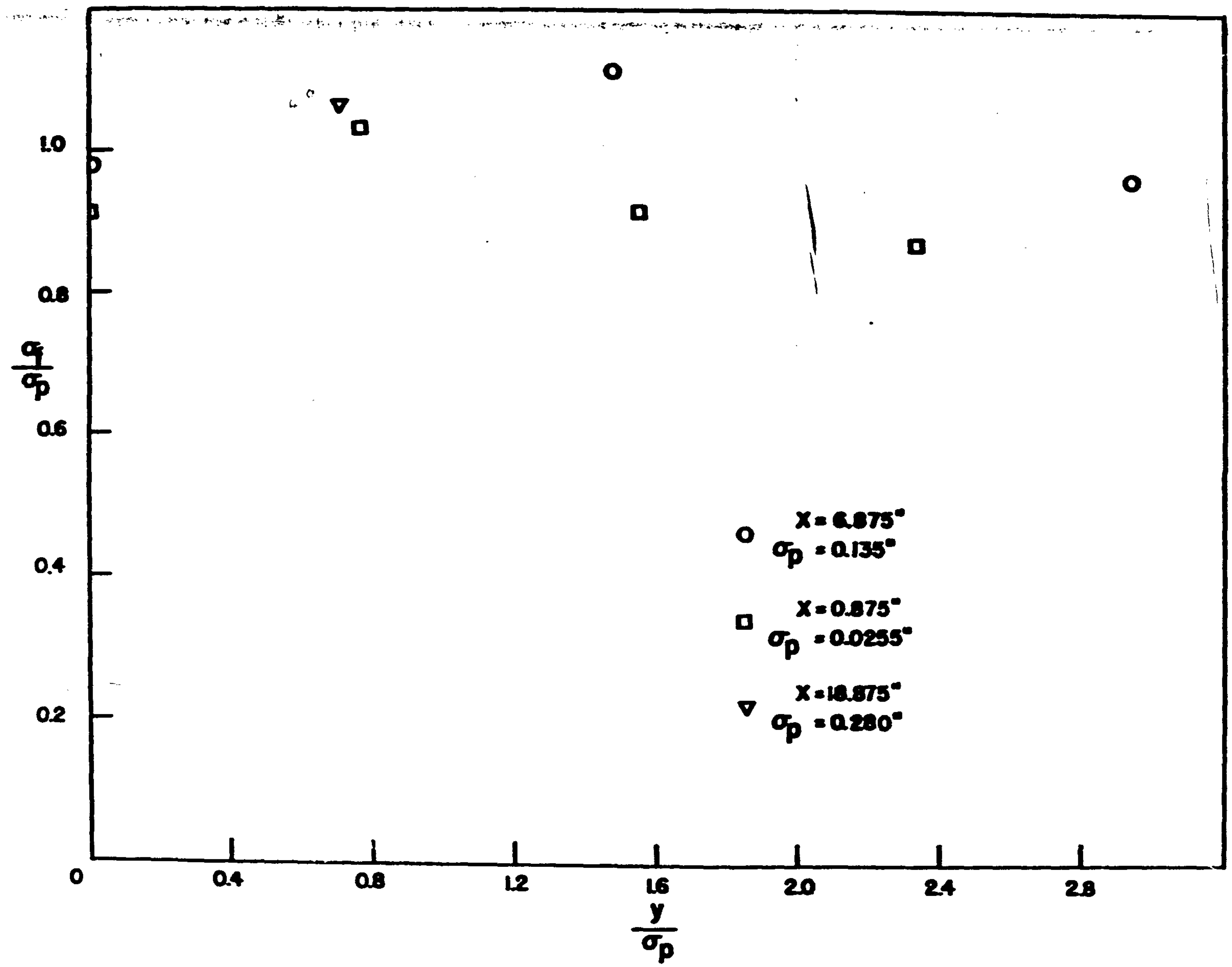


Fig. 5



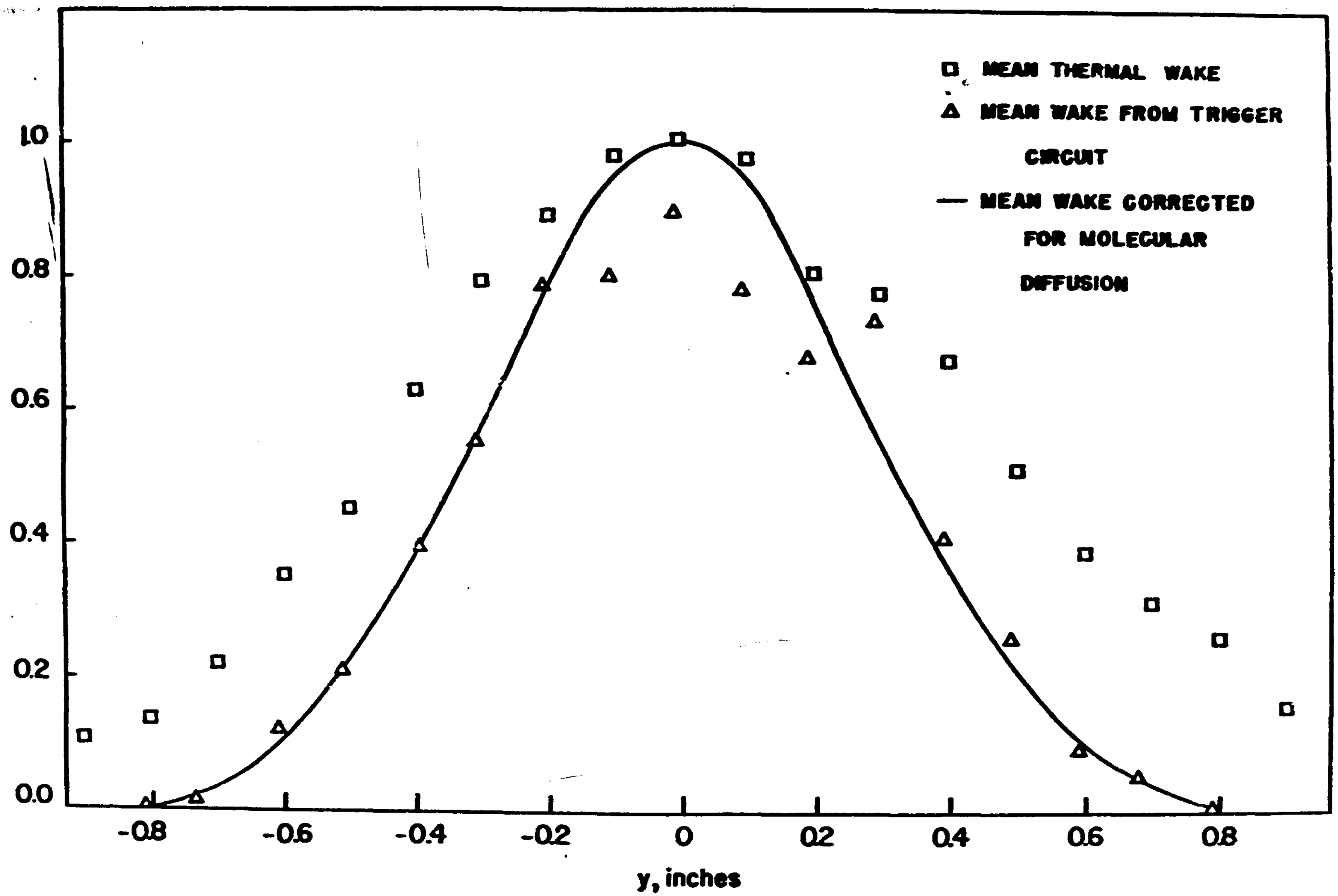
X = 18.875"
y = 0.2"
$\frac{\overline{f(y)\,f(y+r)}}{f^2(0)}$
0.6
0.4
0.2
0
-0.8
-0.4
0
0.4
0.8
r, inches

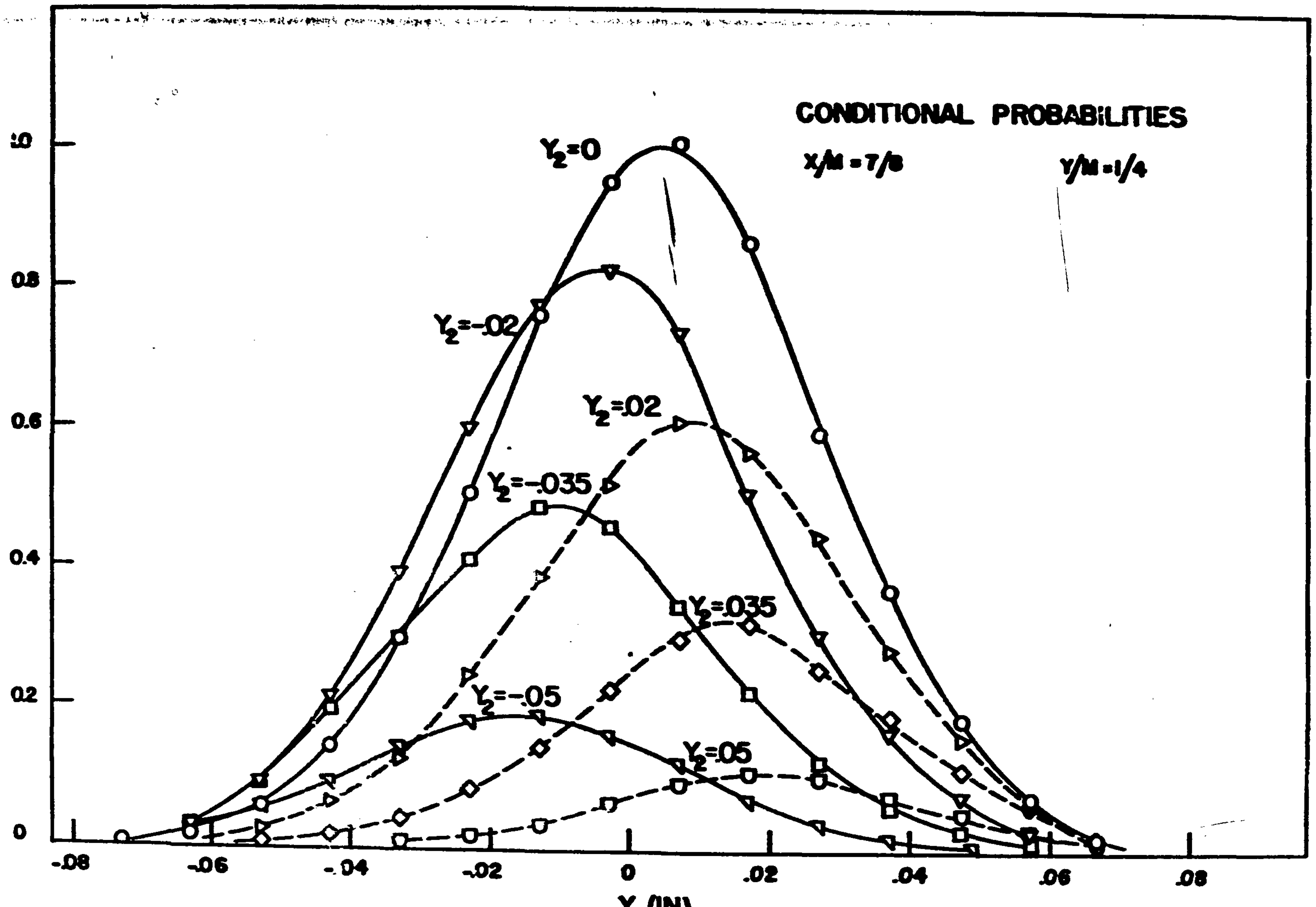
Fig. 6



1.0
0.8
$\frac{\sigma_1}{\sigma_p}$
0.6
0.4
0.2
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
$\frac{y}{\sigma_p}$
X = 6.875"
σ_p = 0.135"
X = 0.875"
σ_p = 0.0255"
X = 18.875"
σ_p = 0.280"

Fig. 7


MEAN THERMAL WAKE
MEAN WAKE FROM TRIGGER CIRCUIT
MEAN WAKE CORRECTED FOR MOLECULAR DIFFUSION
1.0
0.8
0.6
0.4
0.2
0.0
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
y, inches
Fig. 8



CONDITIONAL PROBABILITIES
X/M = 7/8
Y/M = 1/4
Y_2=0
Y_2=-.02
Y_2=.02
Y_2=-.035
Y_2=.035
Y_2=-.05
Y_2=.05
1.0
0.8
0.6
0.4
0.2
0
-.08
-.06
-.04
-.02
0
.02
.04
.06
.08
Y_1 (IN)

Fig. 9

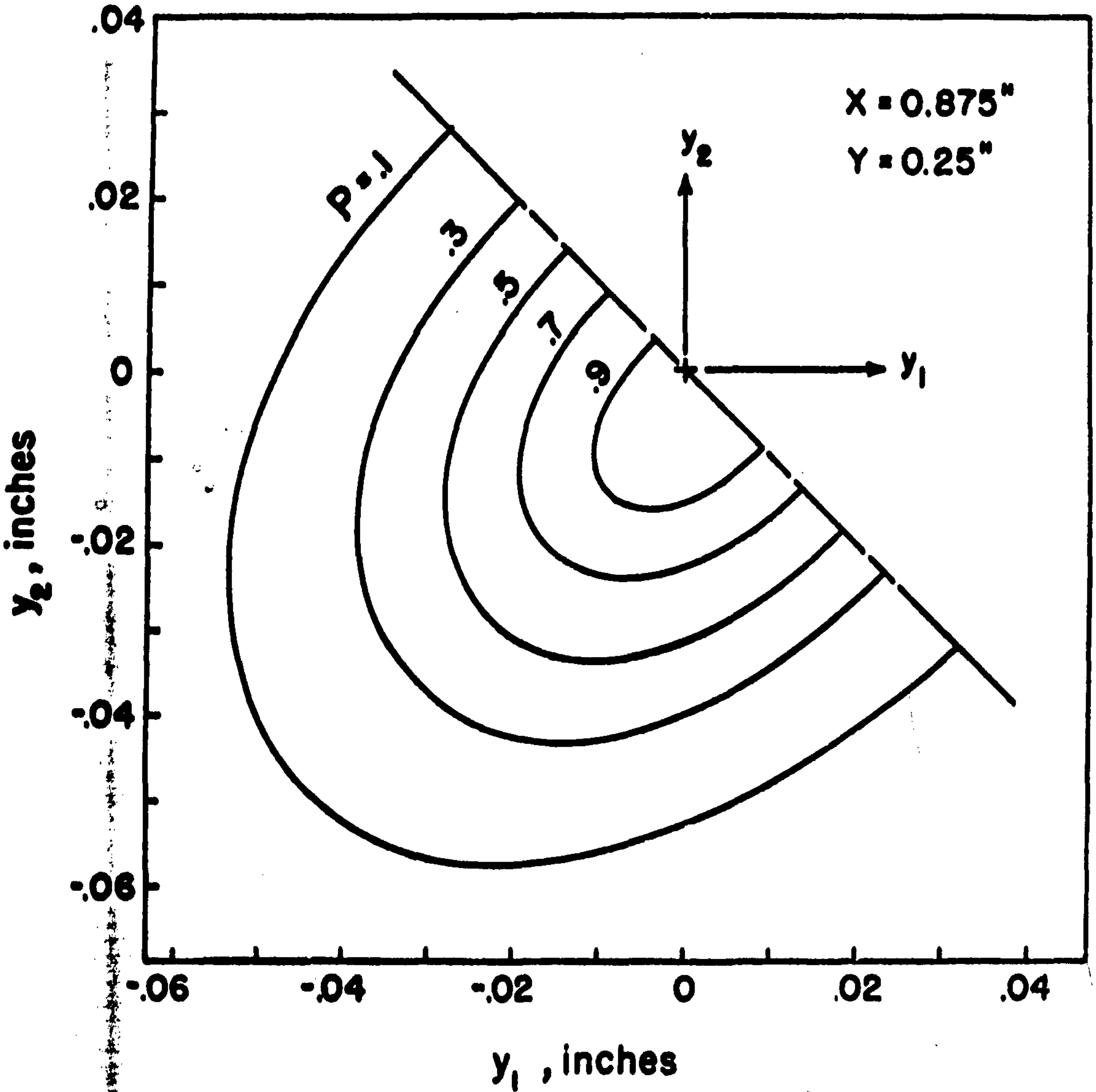


Fig. 10

JOINT PROBABILITY DENSITY

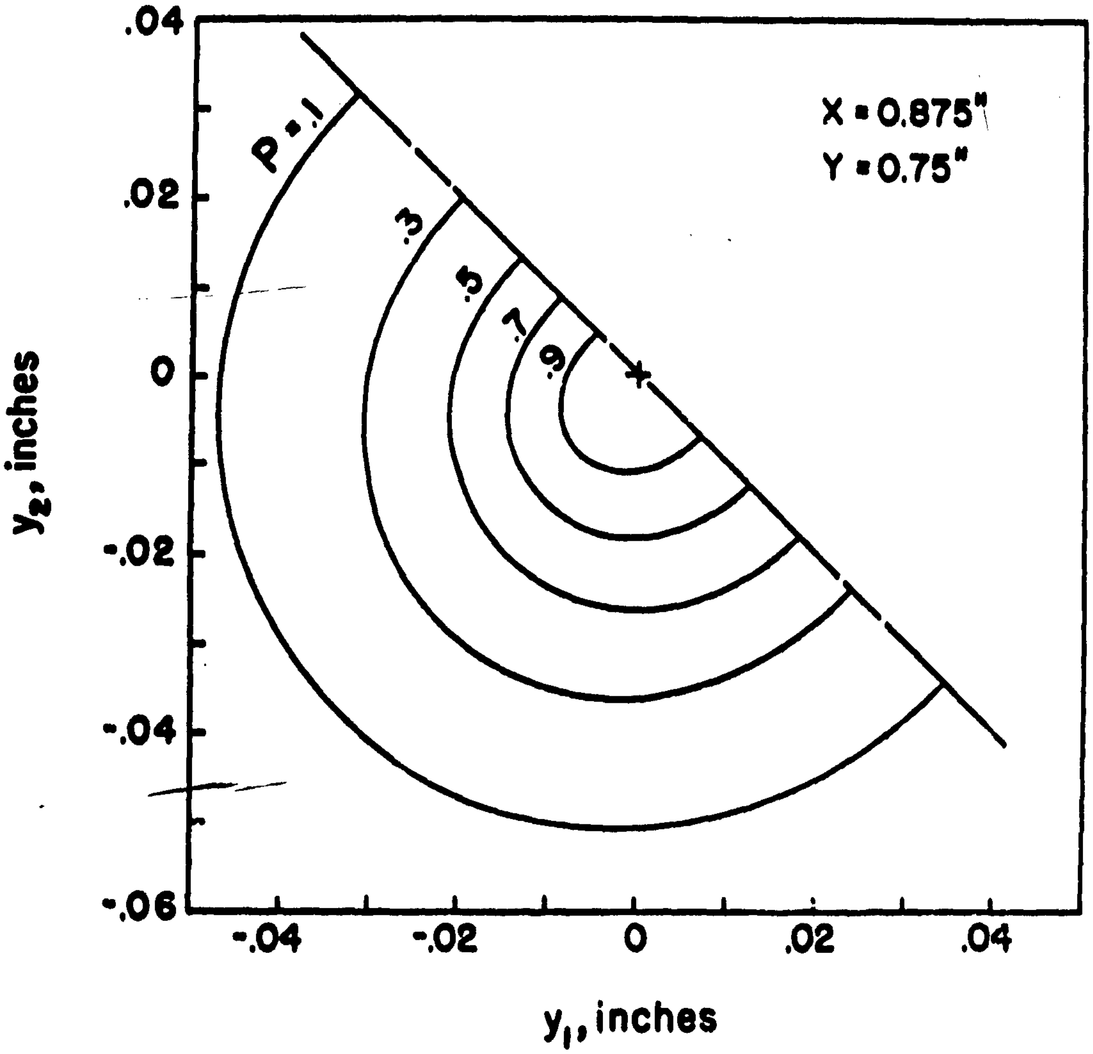


Fig. 11

JOINT PROBABILITY DENSITY

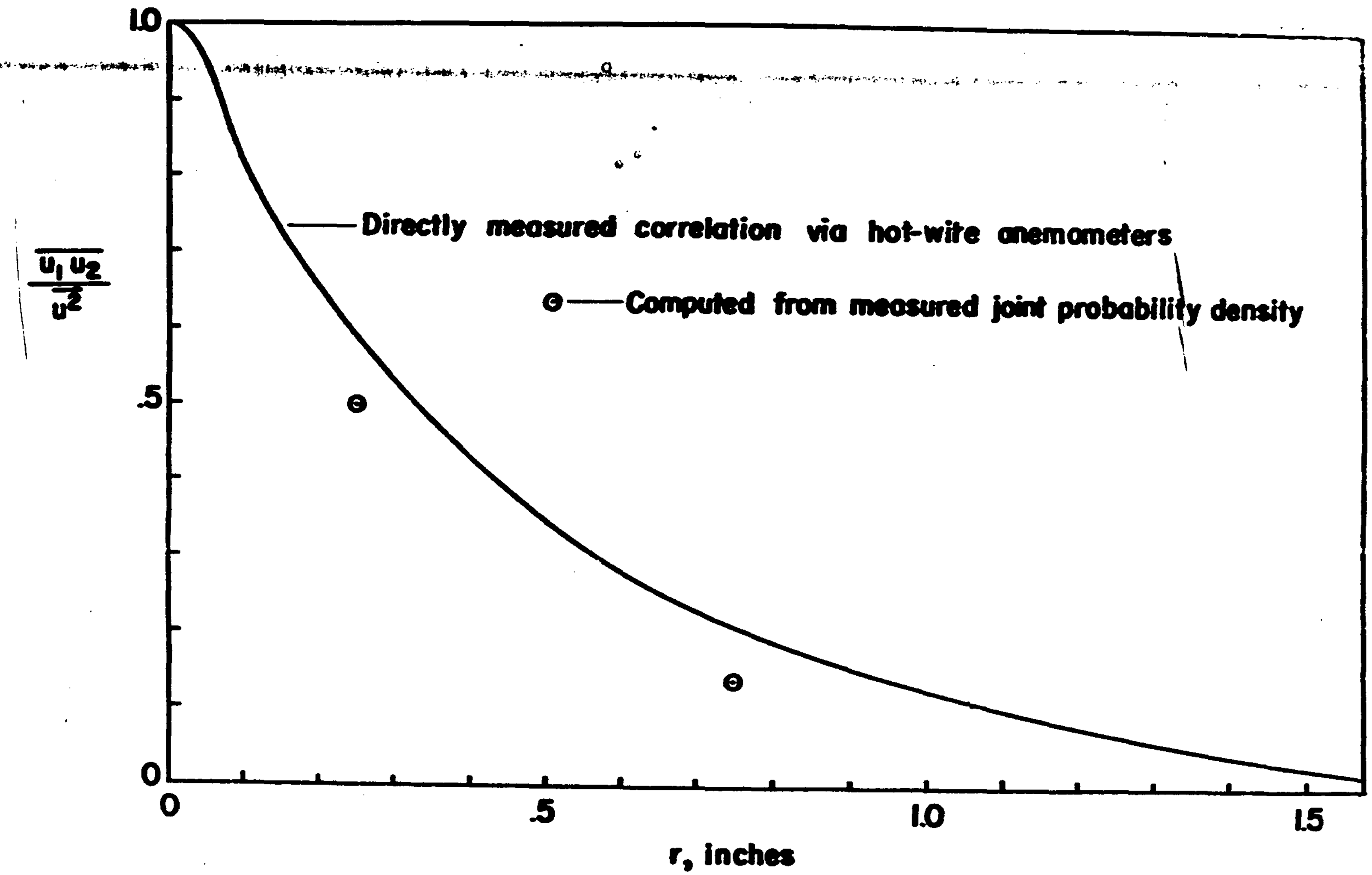


Fig. 12

EULERIAN CORRELATION FUNCTION 20 MESH LENGTHS FROM GRID

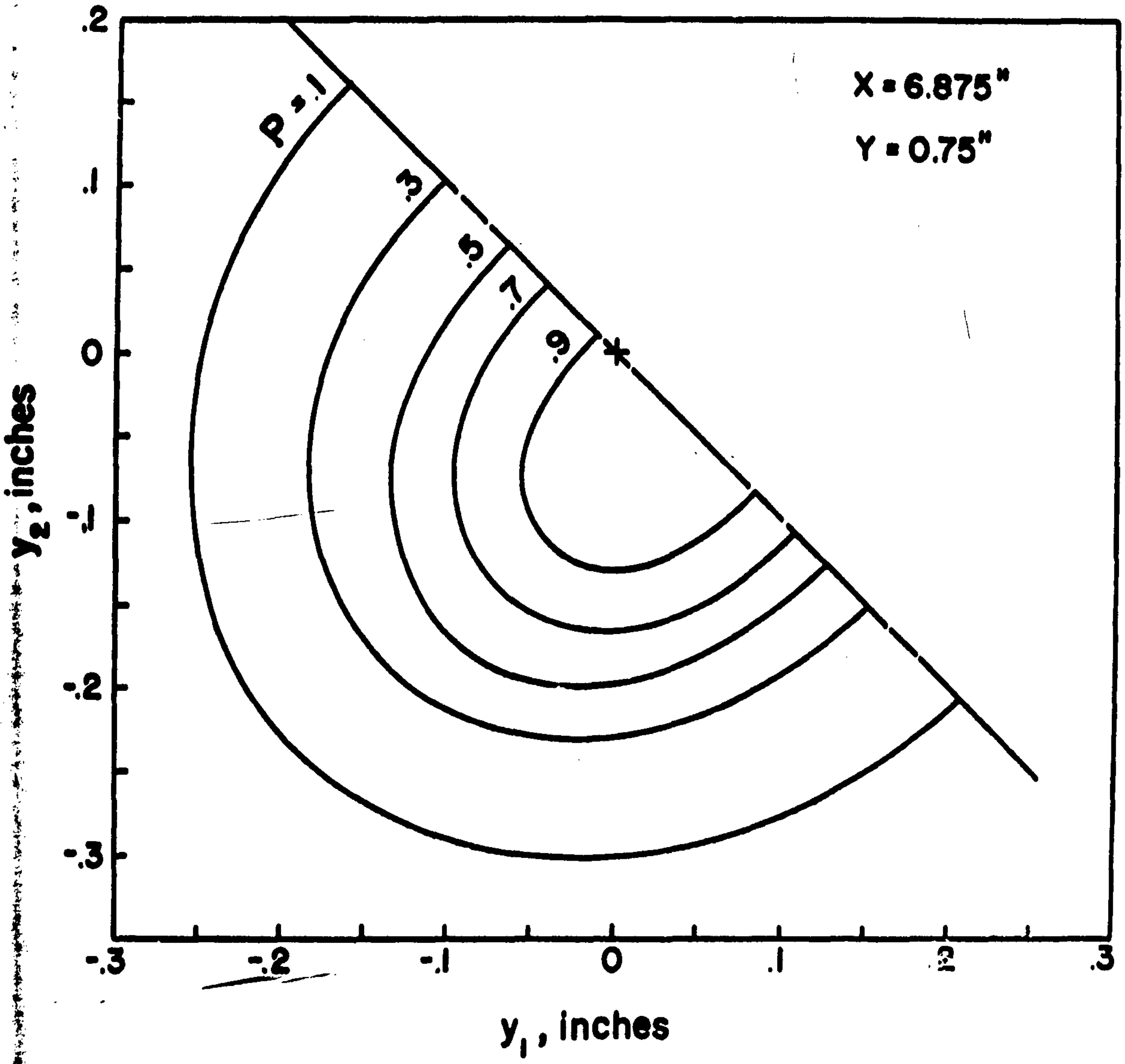


Fig 13
JOINT PROBABILITY DENSITY

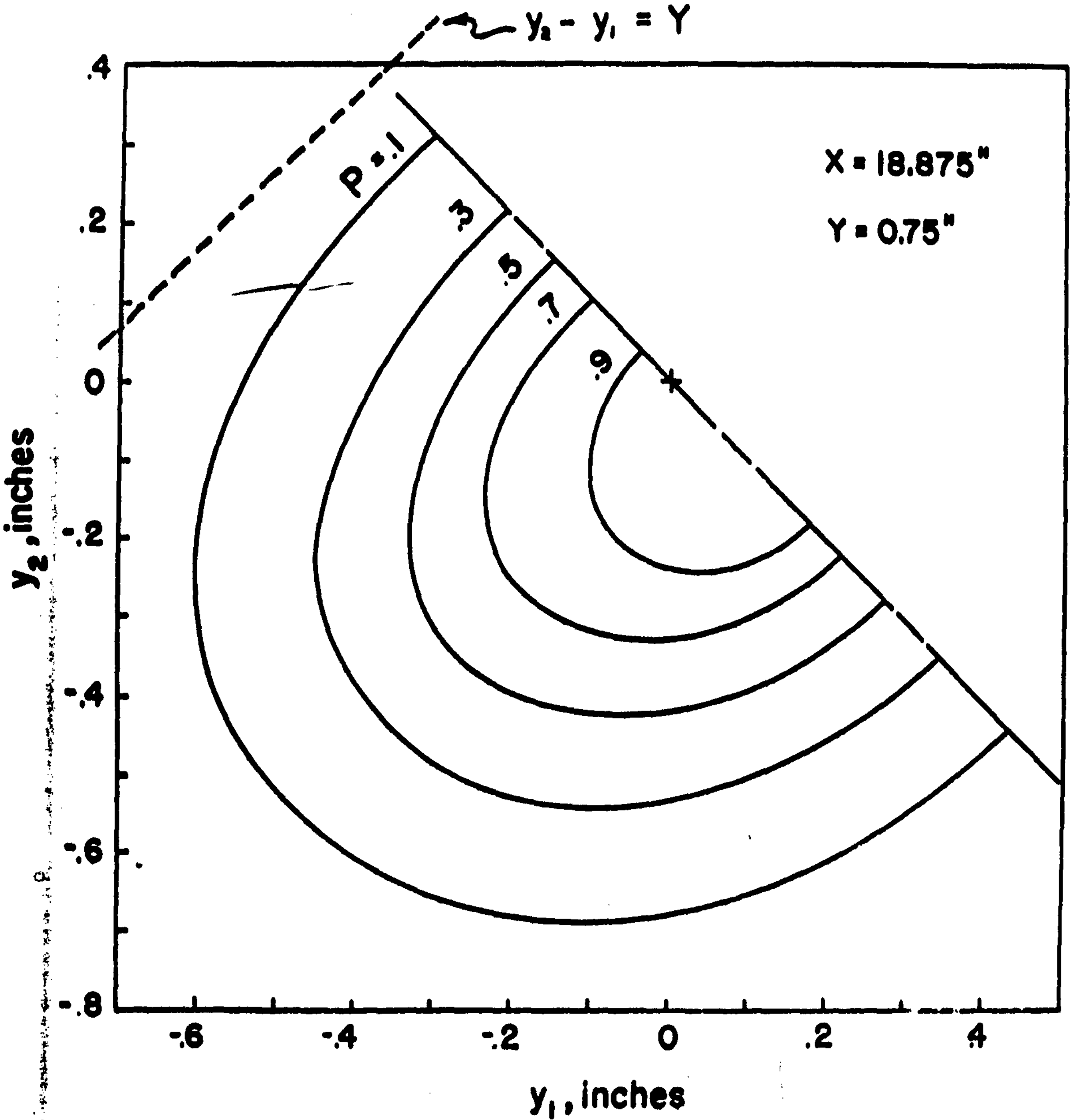
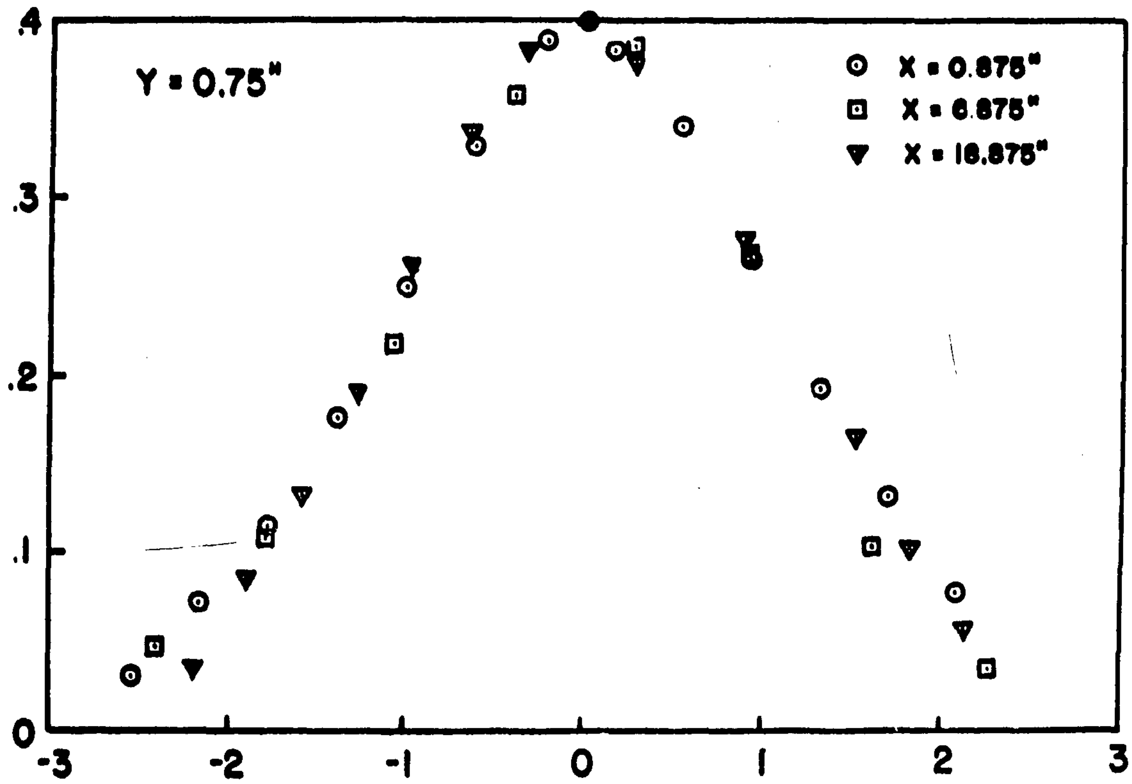


Fig. 14

JOINT PROBABILITY DENSITY



$$\frac{y_1 - y_2}{\sqrt{\overline{(y_1 - y_2)^2}}}$$

Fig. 15

MARGINAL PROBABILITY DENSITIES

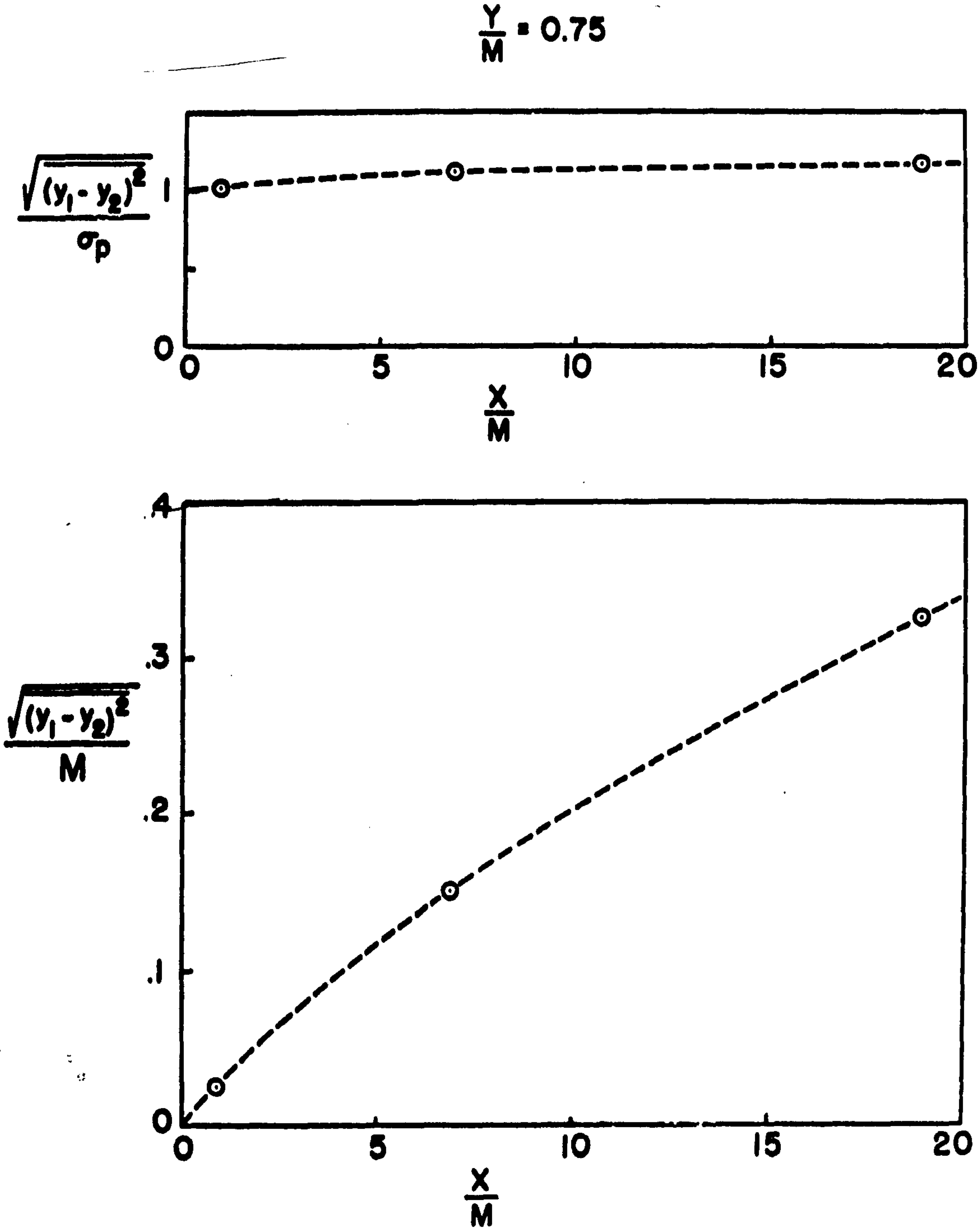


Fig. 16

RELATIVE DISPERSION